

Hong Kong Baptist University
Faculty of Science
Department of Mathematics

Title (Units): MATH3990/MATH7130 Advanced Topics in Mathematics (3,3,0)

Subtitle: Special Topics in Numerical PDEs

Course Aims: This course provides a basic introduction to high order numerical methods for differential equations. Particular attentions will be given to spectral methods which yield extremely high order of numerical accuracy. We emphasize the Chebyshev and Fourier special methods, with their applications to boundary value problems, wave equations and integral equations. The important role of the Fast Fourier Transform (FFT) will be introduced.

Prepared by: Tao Tang

Course Intended Learning Outcomes (CILOs):

Upon successful completion of this course, students should be able to:

No.	Course Intended Learning Outcomes (CILOs)
1	Understand the basic models and derivations for some basic numerical methods
2	Understand the numerical methods for solving BVPs and time-dependent PDEs
3	Learn the basic principle and techniques of the spectral methods
4	Study the tools for efficiency enhancement, including FFT, Rung-Kutta methods, and relevant transformations

Teaching & Learning Activities (TLAs)

CILO	TLAs will include the following:
1-4	Lecture Lectures will be used to introduce the subjects of the course's materials, and examples will be given to aid the learning of the subjects.
2-4	Programming assignment Programming assignments allow students to apply the various techniques learned in the lecture, and help students fully understand them.
1-4	In-home exercises and assignments Homework assignments allow students to measure their progress in learning the subjects of the course.

Assessment:

No.	Assessment Methods	Weighting	CILO Addressed	Remarks
1	Continuous Assessment	40%	1,2,3,4	Homework and programming assignments are designed to measure how well the students have learned various basic concepts in theory of PDEs and their numerical methods, and implement the methods in programming languages.
2	Final Examination	60%	1,2,3,4	Final examination is designed to see how far students have achieved their intended learning outcomes especially in the Knowledge domain. Students should have a thorough understanding on basic PDEs and their numerical methods and being able to analyze them for stability and convergence.

Course Intended Learning Outcomes and Weighting:

Content	CILO No.	Teaching (in hours)
I. Introduction	1	2
II. Review of the finite difference method and the finite element	1,2	6
III. Chebyshev methods	3	8
IV. Fourier spectral methods	3	6
V. Applications to various types of PDEs	3	8
IV. Efficiency enhancement	4	9

Software: Matlab or other programming languages

References: Jie Shen and Tao Tang, "Spectral and High-order Methods with Applications", Science Press, 2006.
Nick Trefethen, "Spectral Method in MATLAB", SIAM, 2000.

Course Content in Outline:	Topic hours:
I. Introduction	2 hours
A. Motivation	
B. Discrete methods for PDEs	
II. FDMs and FEMs	6 hours
A. Consistency	
B. Stability and convergence	
C. Theoretical issues	
III. Chebyshev special methods	8 hours
A. Basic idea	
B. Differentiation matrix	
C. Condition number, spectral radius	
D. Convergence	
IV. Fourier spectral methods	6 hours
A. Basic idea	
B. Differentiation matrix	
C. Condition number, spectral radius	
D. Convergence	
E. FFT	
V. Applications to PDEs	8 hours
A. BVPs	
B. Integral equations	
C. Wave equations	
VI. Efficiency enhancement	9 hours
A. FFT for multi-dimensions	
B. Transformations	
C. Enlarge time steps	